# Additional Notes Oligopoly

## 1 Introduction

We have studied in class the two extreme cases of:

- Many firms in the market (price takers): perfect competition.
- Only one firm in the market (with full power to fix price): monopoly.

It between this two extreme cases (what we observe more frequently in real life) we have **oligopoly**.

**Definition. Oligopoly** is a market with few sellers (offering similar products) that interact strategically

- We study the strategical interaction of firms using tools as for example game theory.
- By Strategic behavior we mean situations in which an individual, when choosing among different actions needs to consider how others might respond to the action he takes.
- In an oligopoly, since all firms have some market power (they can affect the price), profits not only depend on how much each firm produce but also on how much others produce. (This is the outcome is determined by joint strategies).

**Definition. Duopoly:** a special type of oligopoly, only with two firms in the market.

## 2 Duopoly and Nash Equilibrium

Suppose that we have only two firms in the market, if they **collude** (agree upon quantities and prices) and form a **cartel**, they would charge the monopoly price and sell together the monopoly quantity (they also need to split the quantity among them).

- **Alternatively we can say that the profit maximizing price and combined quantity of output when firms decide to work together is the price and quantity of monopoly (recall we find the quantity from MR=MC and recover price from D curve).**

*Remark* 1. Remark: there might be incentives for deviating from the collusion agreement. If for example one firm deviates alone and produce a higher quantity it might get higher profits at the expense of reducing the other firm's profit. So to maintain this agreement there has to be one form of commitment device. (Also as the number of firms in the market grows agreement becomes more difficult)

If on the other hand each firm acts alone, but as we explained above, consider the actions of the other firm, the outcome will be a price that is higher than perfectly competitive price but lower than monopoly and accordingly a total quantity less than the perfectly competitive one but higher than the monopoly one.

- See example of book (pg. 353)
- When each firm1 takes into account firm's 2 quantity and produce accordingly and viceversa for firm 1 then we have a Nash equilibrium.

  **Definition. Nash equilibrium**: situation where individuals choose their best strategy given the strategies others have chosen.

- Example of the book: if firm1 chooses q=40 then firm2 will choose q=40 and viceversa: if firm2 chooses q=40 then firm1 will choose q=40 so none of the firms have incentive to deviate and this is a Nash equilibrium.

  *Remark* 2. Note in a Nash equilibrium no firm has incentive to deviate (contrast this with remark 1 in the collusion explanation).

As an oligopoly grows larger and firms loose market power the outcome approaches to perfect competition.

# 3 Game theory and the Payoff Matrix

## 3.1 Introduction to game theory

Elements of a game:

- players
- actions
- strategies: actions that players take in response to the actions of other players

Example (a simple game)

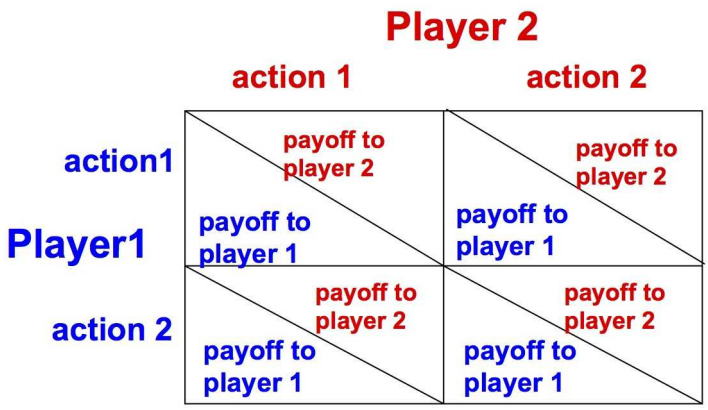
- Players: tom and jerry
- actions: tom can move up or down, jerry can move left or right.
- strategies: we need to specify what action is tom going to chose for each possible action jerry can choose and viceversa
  - example: tom will move up if jerry moves left and will move down when jerry moves right. Likewise jerry will move right if tom moves left and will also move right if tom moves right. (Note right strategy - of jerry- is aka dominant strategy)
  - we actually need to look at the payoff matrix to determine the optimal strategies of each player (see what is more convenient for each of them, given the actions of the other)

Strategical Games

- When players recognize that the outcome of the game is determined not only by their own action but also by the action of other players, we are talking about strategical games (the type of games we will study in this class, and also the type of game that firms in oligopolies are playing).
- Because the outcome depends on the action of both players we use a device (payoff matrix) to summarize the game and to find optimal strategies and equilibriums.

## 3.2 Payoff matrix:

- In the first column and first row we have the name of each player and the actions they can play. Inside of each possible action combination we have the payoff each player will get if those action combination is played.



Back to our simple example

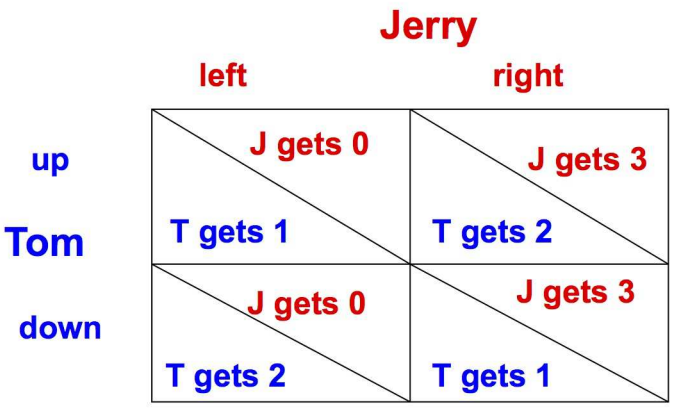
- Player 1: Tom (with actions up and down), player 2: Jerry (with actions left and right)
- Suppose the game goes as follows:
  - Jerry only likes playing right and gets utility of 3 when he manages to do so
  - Tom: prefers to play up if jerry moves right and to move down if jerry moves left

The following payoff matrix depict the described game note the elements:

- in the first column (blue) we have tom and his actions, in the first row (red) Jerry and his actions
- next for each combination of actions we have the payoffs inside of each box for example if tom moves up and jerry moves left then jerry gets 0 and tom gets 1 (upper left corner)

*Remark.* How to read the payoff matrix:

- if Tom decides to play up he would get 1 if Jerry plays left and will get 2 if jerry plays right (this is reading the first row)
- if tom decides to play down he would get 2 if Jerry plays left and 1 if jerry plays right (this is reading the second row) and so on.

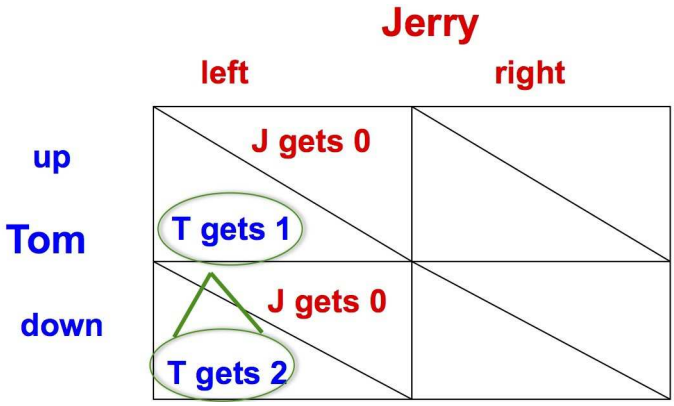



If we want to know what is going to be the outcome of the game (equilibrium strategy and equilibrium payoffs) we first need to get optimal strategies

**How to derive optimal strategies:**

Lets first derive the strategy of Tom, we need to know what he would choose to do if Jerry plays up and what he would choose to do if Jerry plays left:

- If Jerry plays Right (we are only interested in the left column of payoffs - where jerry is actually playing left)
- We need to compare the payoff that Tom will get if he plays up or down

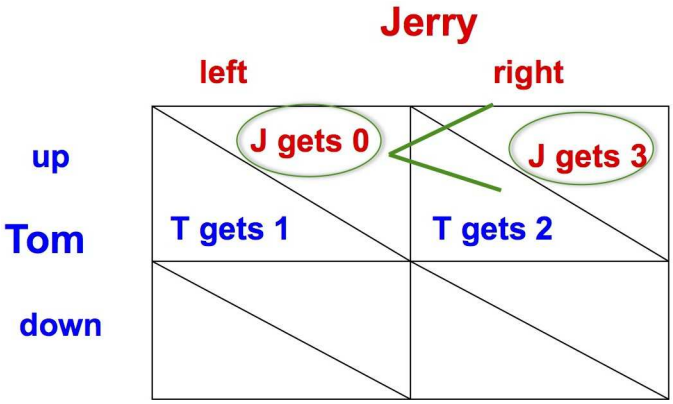



- Focus on Tom's payoffs, given that jerry plays right he will get 1 if he plays up and 2 if he plays down, so he prefers to get 2 (utility)
- Similarly if we now focus on the right column and only on Tom's payoffs if he plays up he gets 2 and if he plays down he gets 1 so he prefers up when jerry plays left

Therefore Tom's strategy is: play down if jerry plays right and play up if jerry plays left.

We can repeat the same analysis to derive what is the best action for jerry to take when tom plays up (so we focus on red payoffs of the first row in the box)

- if tom plays up jerry can play left and get 0 or right and get 3 so he preferrers to play right



- if we repeat the analysis just looking at the second row and red payoffs, when tom plays down jerry prefers right as well

So Jerry strategy is to play right if tom plays up and also play right if Tom plays right

- Note is Right is a dominant strategy, its played no matter what others do.

**Getting the Equilibrium**

- Recall we defined Nash Equilibrium as a situation where individuals choose their best strategy given the strategies others have chosen. This is the same as saying each player is choosing their optimal strategy and no one has incentive to deviate, so lets double check:
  - Jerry is going to play right no matter what
  - Given that Jerry plays right the optimal strategy for Tom is up
  - Double-check if tom plays up then jerry plays right
- So Nash equilibrium strategy is Jerry plays right and Tom plays Up
- The payoffs are Jerry gets 3, Tom gets 2.

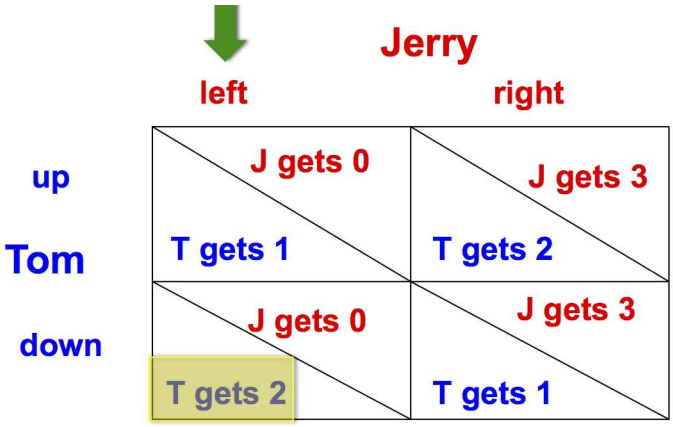
**Remarks on getting the Equilibria**

- There are some games (when we don't have a dominant strategies) that might have more than one Nash equilibria. There are games like the prissioner dilemma where the pareto efficient outcome (the one that maximizes players payoff) is not sustainable without a external cooperation device. We will talk about this in class

## 3.3 Shortcut for getting equilibria:

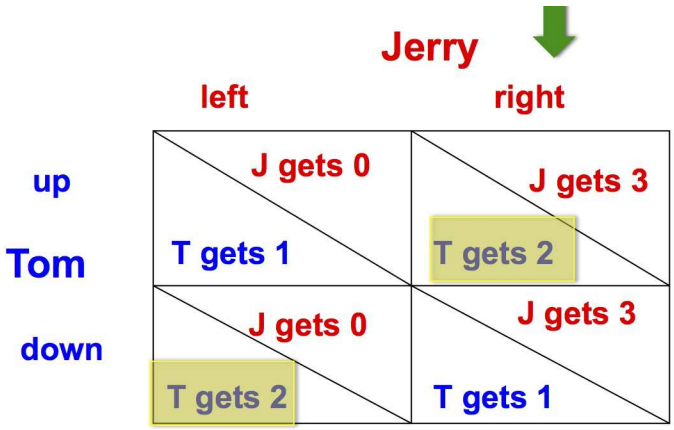
This is just a graphical shortcut for getting the equilibrium we are deriving above. Steps:

1. Take player 1 ( in this case Tom) and get his optimal action given that player two is playing his first action (in this case jerry is playing left)
   - So focus on blue payoffs first column, tom decides to play down, underline the payoff of this player in that entrance




2. Take again player 1 ( in this case Tom) and get his optimal action given that player two is playing his second action (in this case jerry is playing right)

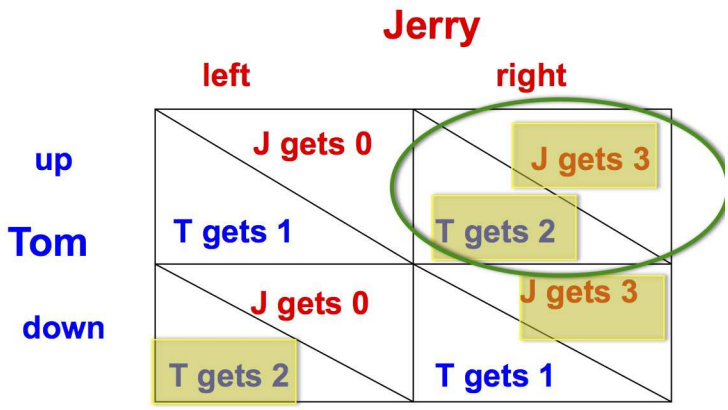
   - So focus on blue payoffs second column, tom decides to play up, underline the payoff of this player in that entrance




3. Now take player 2 ( in this case jerry) and get his optimal action given that player 1 is playing his first action (in this case tom is playing up)

   - So focus on red payoffs row column, jerry decides to play right, underline the payoff of this player in that entrance
   - So focus on red payoffs second column, jerry decides to play right, underline the payoff of this player in that entrance

4. Look at the picture and the equilibrium is the one with two payoffs underlined: jerry plays right tom plays up



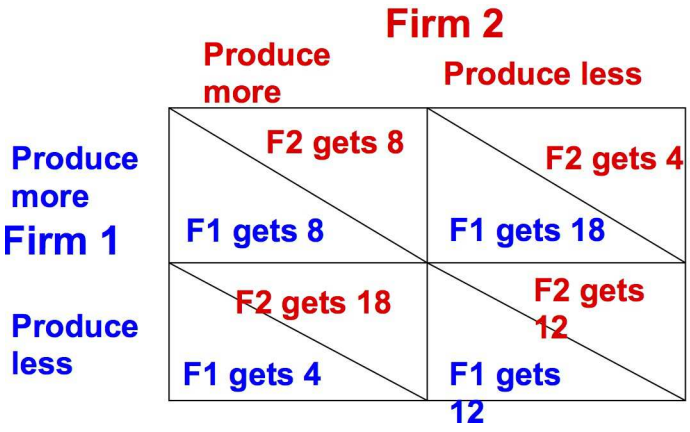

This algorithm and graphical depiction is also useful to double-check that we are actually getting the combination of actions that are optimal for both (intersection of optimal strategies both are underlined!)

# 4 Game Theory and Oligopoly: Using the payoff matrix

Once we understand how the payoff matrix works and how it use it to get the Nash Equilibrium we can apply game theory to Oligopoly.

- When firms collude and act as a cartel we simple have the monopoly outcome
- When on the other hand firms act independently but like strategic players that acknowledge that the actions of others we can derive the equilibrium strategies and outcome of this interaction

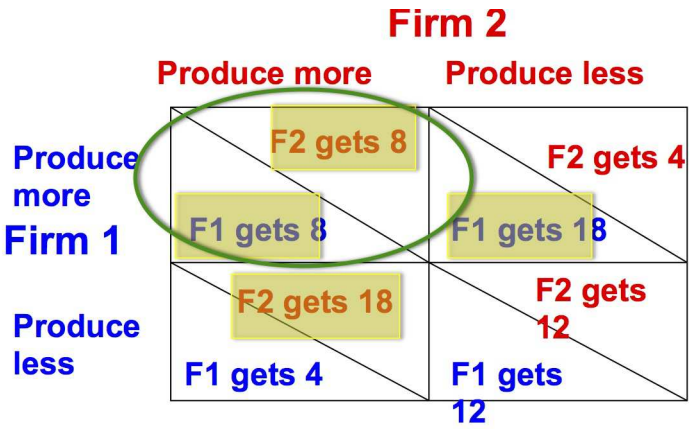
Example: Suppose that firm 1 and firm 2 should decide between producing more or less with the following payoff matrix, (numbers in the matrix are profits)



If firms act independently then the outcome will be the Nash Equilibrium, so following the procedure described above:

- If firm 2 produces more, firm 1 produces more (8>4), if firm 2 produces less then firm 1 produce more (18>12)
- If firm 1 produces more, firm 2 produces more (8>4), if firm 1 produces less then firm 2 produce more (18>12)

So intersection of optimal actions (see figure) is both producing more and the outcome is produce more and produce more with payoffs 8 and 8




In this same example if firms collude they would like first to maximize total profit and then split the profit in between them, so we can use the information of the payoff-matrix to see:

- If both produce more total payoff = 8+8=16
- If firm one produce more and the other less total payoff =4+18=22
- If both produce less total payoff =12+12=24

So the best outcome is for both to produce less, so only if the agree on cooperating before playing the game (before competing in the market) they will achieve higher profits (12).

## Summarizing:

- Using our payoff matrix and finding the Nash equilibrium we can get the outcome of strategical behavior (not collusion)
- If firms collude they will maximize total profit so we need to find the action combination that yields the higher join payoff